# Systematic Programming
# Self-Reduction

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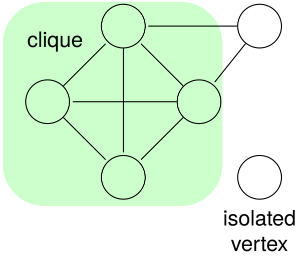
Leibniz Universität Hannover
Institut für Theoretische Informatik

Summer Semester 2020

# Principle of Self-Reduction

- algorithmic technique



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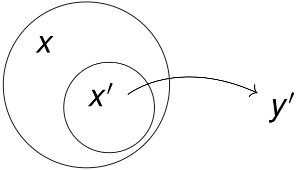
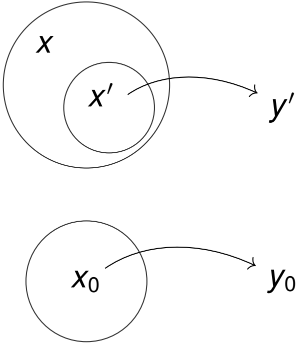


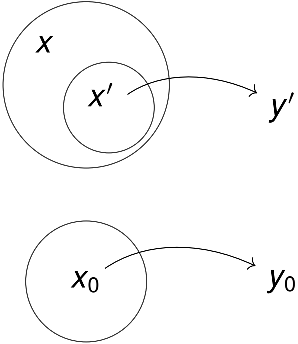
# Principle of Self-Reduction

- algorithmic technique
- case 1: reducible instance $x$
  - find reduced instance $x'$
  - solve $x'$ recursively
  - solve $x$ with $y'$

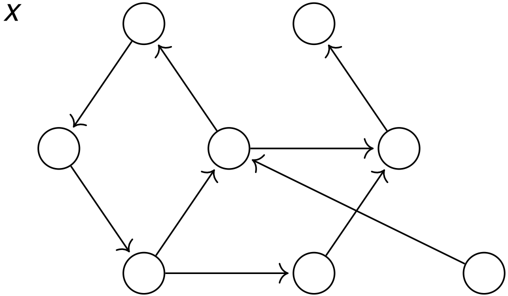


# Principle of Self-Reduction

- algorithmic technique
- case 1: reducible instance $x$
  - find reduced instance $x'$
  - solve $x'$ recursively
  - solve $x$ with $y'$
- case 2: irreducible instance $x_0$
  - solve $x_0$ directly



# Acyclicity of Graphs



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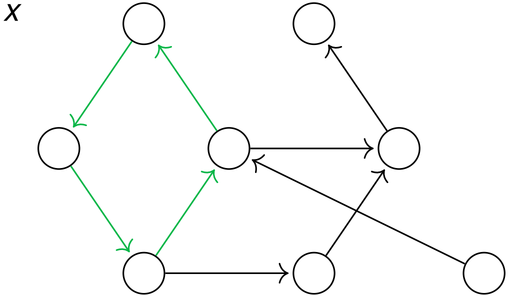

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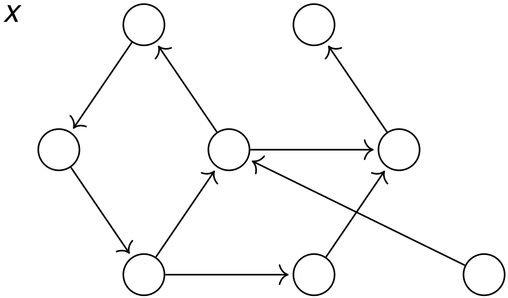

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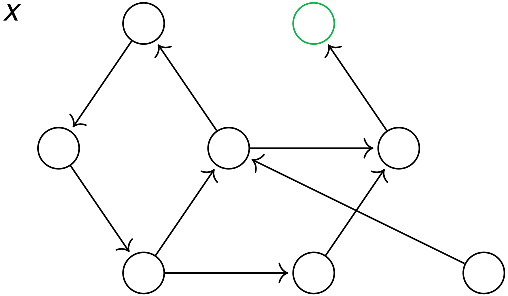

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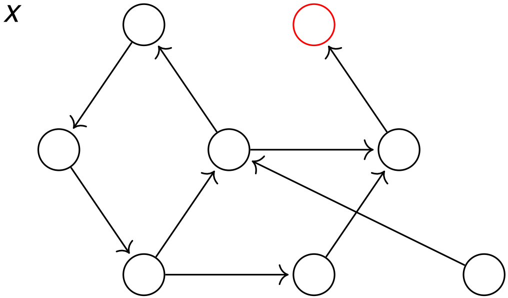

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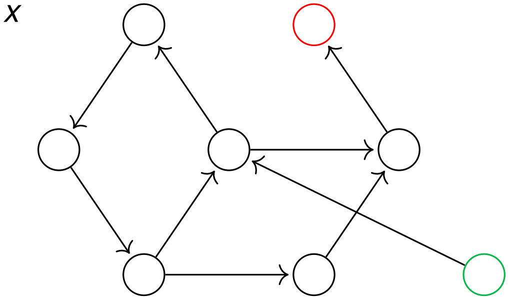

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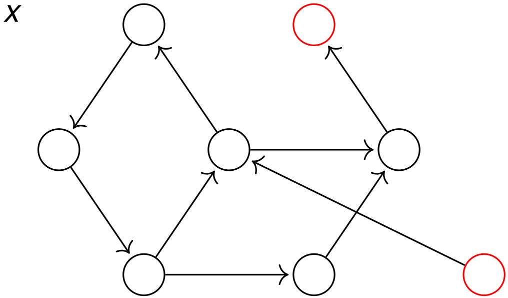

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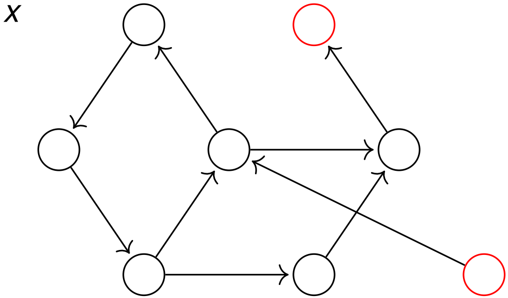


reduction:
remove vertices without incoming
or outgoing edges

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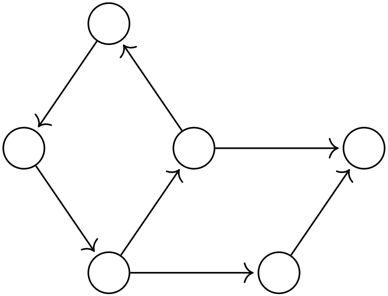

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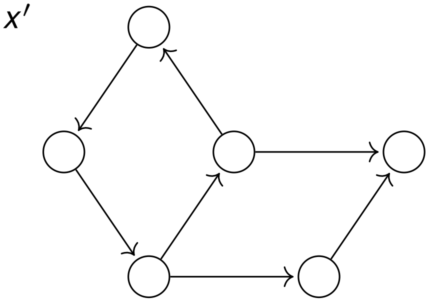


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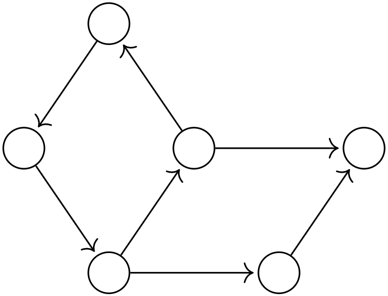
# Acyclicity of Graphs

$x'$



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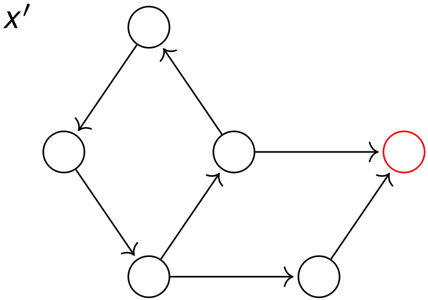


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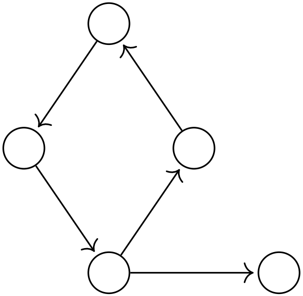

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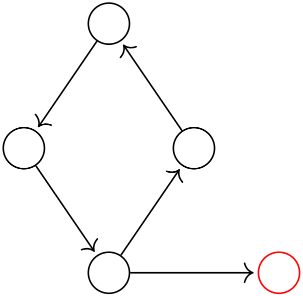

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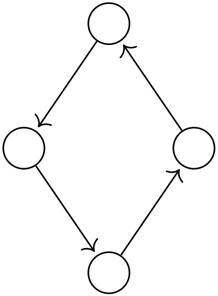

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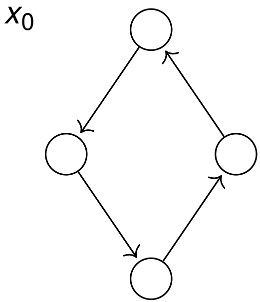


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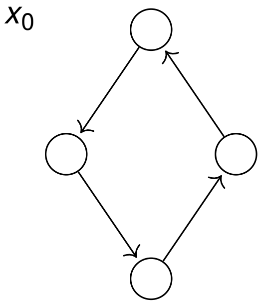


graph acyclic
iff $x_0$ empty

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# Satisfiability of Propositional Formulas

$$F = x_1 \wedge (x_2 \oplus x_1)$$

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-replace first varibale with 0/1
-minimize formula

$F$ satisfiable iff
an irreducible instance $\equiv 1$

# Enumerating Permutations

1 2

# Enumerating Permutations

1 2 3

1 2

# Enumerating Permutations

| 1 | | 2 | | 3 |
|---|---|---|---|---|
| 3 | 1 | | 2 | |

# Enumerating Permutations

1         2         3

3   1       2

$$\text{ins}((1,2),1,3)$$

# Enumerating Permutations

1 2 3

1 3 2

$$\text{ins}((1,2),2,3)$$

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1 2 3

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$\text{ins}((1,2),3,3)$

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| | 2 | | 1 | |

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| | | | | |
|---|---|---|---|---|
| 1 | | 2 | | 3 |
| | 1 | | 2 | |
| | 2 | | 1 | |

$$|p(3)| = 3!$$

# Enumerating Permutations

1 2 3

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2 1

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$$|p(3)| = 3! = 2 \times 3$$

$$p(3) = \{\text{ins}(x, i, 3) \mid x \in p(2), 1 \leq i \leq 3\}$$

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$$|p(3)| = 3! = 2 \times 3$$

$$p(n) = \{\text{ins}(x, i, n) \mid x \in p(n-1), 1 \leq i \leq n\}$$
$$p(1) = \{(1)\}$$