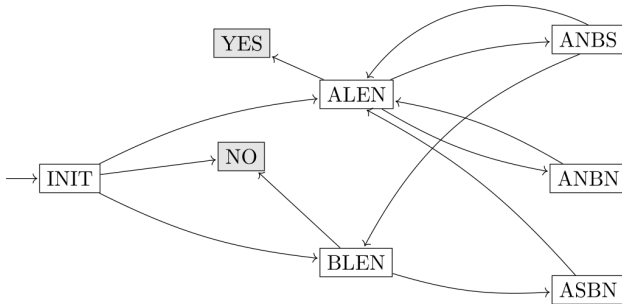


Trace-Based Programming Method



Confidential String Matching

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

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Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

Question: $a_1 \dots a_n = b_1 \dots b_m?$

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

Question: $a_1 \dots a_n = b_1 \dots b_m?$

Example: $A = [ab, cd, e], B = [a, bcd, e]$

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

Question: $a_1 \dots a_n = b_1 \dots b_m?$

Example: $A = [ab, cd, e], B = [a, bcd, e]$

→ abcde ✓

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

Question: $a_1 \dots a_n = b_1 \dots b_m?$

Example: $A = [??, ??, ?], B = [?, ???, ?]$

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

Question: $a_1 \dots a_n = b_1 \dots b_m?$

Example: $A = [??, ??, ?], B = [?, ???, ?]$

$\text{substr}(A[1], 0, 2) = \text{substr}(B[2], 1, 2)$

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

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$\text{substr}(A[1], 0, 2) = \text{substr}(B[2], 1, 2)$


index

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

Question: $a_1 \dots a_n = b_1 \dots b_m?$

Example: $A = [??, ??, ?], B = [?, ???, ?]$

$\text{substr}(A[1], 0, 2) = \text{substr}(B[2], 1, 2)$

index offset

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

Question: $a_1 \dots a_n = b_1 \dots b_m?$

Example: $A = [??, ??, ?], B = [?, ???, ?]$

$\text{substr}(A[1], 0, 2) = \text{substr}(B[2], 1, 2)$

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Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

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Example: $A = [??, ??, ?], B = [?, ???, ?]$

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Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

Question: $a_1 \dots a_n = b_1 \dots b_m?$

Example: $A = [??, ??, ?], B = [?, ???, ?]$

$\text{substr}(A[1], 0, 2) = \text{substr}(B[2], 1, 2)$

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Algorithm:

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

Question: $a_1 \dots a_n = b_1 \dots b_m?$

Example: $A = [??, ??, ?], B = [?, ???, ?]$

$\text{substr}(A[1], 0, 2) = \text{substr}(B[2], 1, 2)$

index offset length

Algorithm:

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

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index offset length

Algorithm:

<i>A</i>	c	e		c	e	c	e	c	e	c	e
<i>B</i>	c	e		c	e	c	e	c	e	c	e

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Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

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Example: $A = [??, ??, ?], B = [?, ???, ?]$

$\text{substr}(A[1], 0, 2) = \text{substr}(B[2], 1, 2)$

index offset length

Algorithm:

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

$\text{substr}(A[1], 0, 2) \stackrel{?}{=} \text{substr}(B[1], 0, 2)$

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

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$\text{substr}(A[1], 0, 2) = \text{substr}(B[2], 1, 2)$

index offset length

Algorithm:

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

$\text{substr}(A[1], 0, 2) = \text{substr}(B[1], 0, 2)$

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

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$\text{substr}(A[1], 0, 2) = \text{substr}(B[2], 1, 2)$

index offset length

Algorithm:

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

$\text{substr}(A[1], 0, 2) = \text{substr}(B[1], 0, 2)$

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

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$\text{substr}(A[1], 0, 2) = \text{substr}(B[2], 1, 2)$

index offset length

Algorithm:

<i>A</i>	c	e		c	e	c	e	c	e	c	e
<i>B</i>	c	e		c	e	c	e	c	e	c	e

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

Question: $a_1 \dots a_n = b_1 \dots b_m?$

Example: $A = [??, ??, ?], B = [?, ???, ?]$

$\text{substr}(A[1], 0, 2) = \text{substr}(B[2], 1, 2)$

index offset length

Algorithm:

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

Question: $a_1 \dots a_n = b_1 \dots b_m?$

Example: $A = [??, ??, ?], B = [?, ???, ?]$

$\text{substr}(A[1], 0, 2) = \text{substr}(B[2], 1, 2)$

index offset length

Algorithm:

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

$\text{substr}(A[1], 2, 2) \stackrel{?}{=} \text{substr}(B[2], 0, 2)$

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

Question: $a_1 \dots a_n = b_1 \dots b_m?$

Example: $A = [??, ??, ?], B = [?, ???, ?]$

$\text{substr}(A[1], 0, 2) = \text{substr}(B[2], 1, 2)$

index offset length

Algorithm:

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

$\text{substr}(A[1], 2, 2) = \text{substr}(B[2], 0, 2)$

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

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Algorithm:

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

$\text{substr}(A[1], 2, 2) = \text{substr}(B[2], 0, 2)$

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

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Algorithm:

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

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Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

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Algorithm:

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

Question: $a_1 \dots a_n = b_1 \dots b_m?$

Example: $A = [??, ??, ?], B = [?, ???, ?]$

$\text{substr}(A[1], 0, 2) = \text{substr}(B[2], 1, 2)$

index offset length

Algorithm:

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

$\text{substr}(A[2], 0, 1) \stackrel{?}{=} \text{substr}(B[2], 2, 1)$

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

Question: $a_1 \dots a_n = b_1 \dots b_m?$

Example: $A = [??, ??, ?], B = [?, ???, ?]$

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A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

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Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

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A	c	e	c	e	c	e	c	e	c	e
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Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

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index offset length

Algorithm:

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

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Example: $A = [??, ??, ?], B = [?, ???, ?]$

$\text{substr}(A[1], 0, 2) = \text{substr}(B[2], 1, 2)$

index offset length

Algorithm:

<i>A</i>	c	e	c	e	c	e		c	e	c	e
<i>B</i>	c	e	c	e	c	e		c	e	c	e

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

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Example: $A = [??, ??, ?], B = [?, ???, ?]$

$\text{substr}(A[1], 0, 2) = \text{substr}(B[2], 1, 2)$

index offset length

Algorithm:

A	c	e	c	e	c	e		c	e	c	e
B	c	e	c	e	c	e		c	e	c	e

$\text{substr}(A[2], 1, 1) \stackrel{?}{=} \text{substr}(B[3], 0, 1)$

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

Question: $a_1 \dots a_n = b_1 \dots b_m?$

Example: $A = [??, ??, ?], B = [?, ???, ?]$

$\text{substr}(A[1], 0, 2) = \text{substr}(B[2], 1, 2)$

index offset length

Algorithm:

A	c	e	c	e	c	e		c	e	c	e
B	c	e	c	e	c	e		c	e	c	e

$\text{substr}(A[2], 1, 1) = \text{substr}(B[3], 0, 1)$

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

Question: $a_1 \dots a_n = b_1 \dots b_m?$

Example: $A = [??, ??, ?], B = [?, ???, ?]$

$\text{substr}(A[1], 0, 2) = \text{substr}(B[2], 1, 2)$

index offset length

Algorithm:

A	c	e	c	e	c	e		c	e	c	e
B	c	e	c	e	c	e		c	e	c	e

$\text{substr}(A[2], 1, 1) = \text{substr}(B[3], 0, 1)$

Confidential String Matching

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$\text{substr}(A[1], 0, 2) = \text{substr}(B[2], 1, 2)$

index offset length

Algorithm:

A	c	e	c	e	c	e	c	e	c	e
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Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

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$\text{substr}(A[1], 0, 2) = \text{substr}(B[2], 1, 2)$

index offset length

Algorithm:

<i>A</i>	c	e	c	e	c	e	c	e		c	e
<i>B</i>	c	e	c	e	c	e	c	e		c	e

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

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Example: $A = [??, ??, ?], B = [?, ???, ?]$

$$\text{substr}(A[1], 0, 2) = \text{substr}(B[2], 1, 2)$$

index offset length

Algorithm:

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

$$\text{substr}(A[3], 0, 2) \stackrel{?}{=} \text{substr}(B[3], 1, 2)$$

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

Question: $a_1 \dots a_n = b_1 \dots b_m?$

Example: $A = [??, ??, ?], B = [?, ???, ?]$

$\text{substr}(A[1], 0, 2) = \text{substr}(B[2], 1, 2)$

index offset length

Algorithm:

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

$\text{substr}(A[3], 0, 2) = \text{substr}(B[3], 1, 2)$

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

Question: $a_1 \dots a_n = b_1 \dots b_m?$

Example: $A = [??, ??, ?], B = [?, ???, ?]$

$\text{substr}(A[1], 0, 2) = \text{substr}(B[2], 1, 2)$

index offset length

Algorithm:

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

$\text{substr}(A[3], 0, 2) = \text{substr}(B[3], 1, 2)$

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

Question: $a_1 \dots a_n = b_1 \dots b_m?$

Example: $A = [??, ??, ?], B = [?, ???, ?]$

$\text{substr}(A[1], 0, 2) = \text{substr}(B[2], 1, 2)$

index offset length

Algorithm:

<i>A</i>	c	e	c	e	c	e	c	e		c	e
<i>B</i>	c	e	c	e	c	e	c	e		c	e

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

Question: $a_1 \dots a_n = b_1 \dots b_m?$

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$\text{substr}(A[1], 0, 2) = \text{substr}(B[2], 1, 2)$

index offset length

Algorithm:

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

Question: $a_1 \dots a_n = b_1 \dots b_m?$

Example: $A = [??, ??, ?], B = [?, ???, ?]$

$\text{substr}(A[1], 0, 2) = \text{substr}(B[2], 1, 2)$

index offset length

Algorithm:

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

$\text{substr}(A[4], 0, 2) \stackrel{?}{=} \text{substr}(B[4], 0, 2)$

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

Question: $a_1 \dots a_n = b_1 \dots b_m?$

Example: $A = [??, ??, ?], B = [?, ???, ?]$

$\text{substr}(A[1], 0, 2) = \text{substr}(B[2], 1, 2)$

index offset length

Algorithm:

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

$\text{substr}(A[4], 0, 2) = \text{substr}(B[4], 0, 2)$

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

Question: $a_1 \dots a_n = b_1 \dots b_m?$

Example: $A = [??, ??, ?], B = [?, ???, ?]$

$\text{substr}(A[1], 0, 2) = \text{substr}(B[2], 1, 2)$

index offset length

Algorithm:

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

$\text{substr}(A[4], 0, 2) = \text{substr}(B[4], 0, 2)$

Confidential String Matching

Input: $A = [a_1, \dots, a_n], B = [b_1, \dots, b_m]$

Question: $a_1 \dots a_n = b_1 \dots b_m?$

Example: $A = [??, ??, ?], B = [?, ???, ?]$

$\text{substr}(A[1], 0, 2) = \text{substr}(B[2], 1, 2)$

index offset length

Algorithm:

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

✓

Write Traces

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Write Traces

$\text{substr}(A[\mathit{ca}], \mathit{oa}, l) = \text{substr}(B[\mathit{cb}], \mathit{ob}, l)$

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Write Traces

ca cb oa ob l r

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e		c	e	c	e	c	e	c	e
<i>B</i>	c	e		c	e	c	e	c	e	c	e

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
				2	

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e		c	e	c	e	c	e	c	e
<i>B</i>	c	e		c	e	c	e	c	e	c	e

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
					2

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e		c	e	c	e	c	e	c	e
<i>B</i>	c	e		c	e	c	e	c	e	c	e

$\text{substr}(A[1], 0, 2) = \text{substr}(B[1], 0, 2)$

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
					2

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e		c	e	c	e	c	e	c	e
<i>B</i>	c	e		c	e	c	e	c	e	c	e

$\text{substr}(A[1], 0, 2) = \text{substr}(B[1], 0, 2)$

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
				2	
	2	2	0		

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
				2	
	2	2	0		

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
				2	
	2	2	0		
				2	

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
				2	
	2	2	0		
				2	

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

$\text{substr}(A[1], 2, 2) = \text{substr}(B[2], 0, 2)$

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
				2	
	2	2	0		
				2	

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

$\text{substr}(A[1], 2, 2) = \text{substr}(B[2], 0, 2)$

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
				2	
	2	2	0		
				2	
2		0	2		

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
				2	
	2	2	0		
				2	
2		0	2		

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
				2	
	2	2	0		
				2	
2		0	2		
				1	

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
				2	
	2	2	0		
				2	
2		0	2		
				1	

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

$\text{substr}(A[2], 0, 1) = \text{substr}(B[2], 2, 1)$

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
				2	
	2	2	0		
				2	
2		0	2		
				1	

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

$\text{substr}(A[2], 0, 1) = \text{substr}(B[2], 2, 1)$

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
				2	
	2	2	0		
				2	
2		0	2		
				1	
	3	1	0		

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
				2	
	2	2	0		
				2	
2		0	2		
				1	
	3	1	0		

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e	c	e	c	e		c	e	c	e
<i>B</i>	c	e	c	e	c	e		c	e	c	e

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
				2	
	2	2	0		
				2	
2		0	2		
				1	
	3	1	0		
				1	

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e	c	e	c	e		c	e	c	e
<i>B</i>	c	e	c	e	c	e		c	e	c	e

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
				2	
	2	2	0		
				2	
2		0	2		
				1	
	3	1	0		
				1	

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

$\text{substr}(A[2], 1, 1) = \text{substr}(B[3], 0, 1)$

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
				2	
	2	2	0		
				2	
2		0	2		
				1	
	3	1	0		
				1	

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

$\text{substr}(A[2], 1, 1) = \text{substr}(B[3], 0, 1)$

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
				2	
	2	2	0		
				2	
2		0	2		
				1	
	3	1	0		
				1	
3		0	1		

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
				2	
	2	2	0		
				2	
2		0	2		
				1	
	3	1	0		
				1	
3		0	1		

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e	c	e	c	e	c	e		c	e
<i>B</i>	c	e	c	e	c	e	c	e		c	e

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
				2	
	2	2	0		
				2	
2		0	2		
				1	
	3	1	0		
				1	
3		0	1		
				2	

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e	c	e	c	e	c	e		c	e
<i>B</i>	c	e	c	e	c	e	c	e		c	e

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
				2	
	2	2	0		
				2	
2		0	2		
				1	
	3	1	0		
				1	
3		0	1		
				2	

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

$\text{substr}(A[3], 0, 2) = \text{substr}(B[3], 1, 2)$

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
				2	
	2	2	0		
				2	
2		0	2		
				1	
	3	1	0		
				1	
3		0	1		
				2	

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e	c	e	c	e	c	e		c	e
<i>B</i>	c	e	c	e	c	e	c	e		c	e

$\text{substr}(A[3], 0, 2) = \text{substr}(B[3], 1, 2)$

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
				2	
	2	2	0		
				2	
2		0	2		
				1	
	3	1	0		
				1	
3		0	1		
				2	
4	4	0	0		

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
				2	
	2	2	0		
				2	
2		0	2		
				1	
	3	1	0		
				1	
3		0	1		
				2	
4	4	0	0		

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
				2	
	2	2	0		
				2	
2		0	2		
				1	
	3	1	0		
				1	
3		0	1		
				2	
4	4	0	0		
				2	

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
				2	
	2	2	0		
				2	
2		0	2		
				1	
	3	1	0		
				1	
3		0	1		
				2	
4	4	0	0		
				2	

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

$\text{substr}(A[4], 0, 2) = \text{substr}(B[4], 0, 2)$

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
				2	
	2	2	0		
				2	
2		0	2		
				1	
	3	1	0		
				1	
3		0	1		
				2	
4	4	0	0		
				2	

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

$\text{substr}(A[4], 0, 2) = \text{substr}(B[4], 0, 2)$

Write Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
				2	
	2	2	0		
				2	
2		0	2		
				1	
	3	1	0		
				1	
3		0	1		
				2	
4	4	0	0		
				2	

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

✓

Write Traces

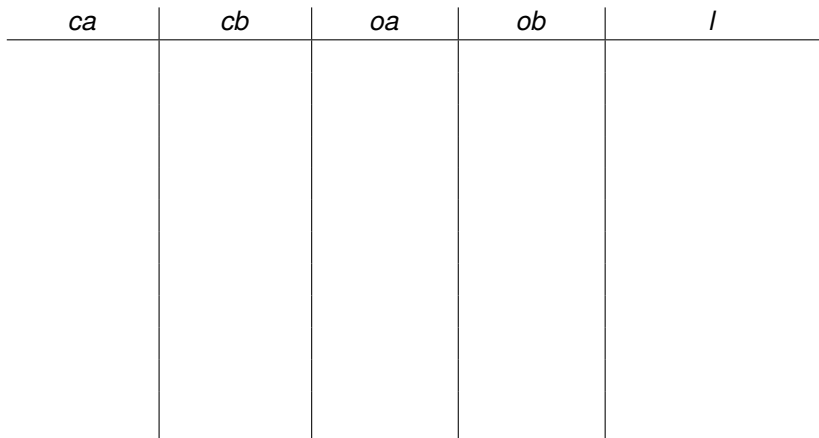
<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>	<i>r</i>
1	1	0	0		
				2	
	2	2	0		
				2	
2		0	2		
				1	
	3	1	0		
				1	
3		0	1		
				2	
4	4	0	0		
				2	
					✓

$\text{substr}(A[\text{ca}], \text{oa}, l) = \text{substr}(B[\text{cb}], \text{ob}, l)$

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

✓

Generalize Traces



A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

Generalize Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>
1	1	0	0	

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

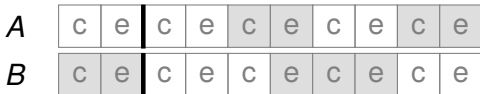
Generalize Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>
1 = 1	1 = 1	0 = 0	0 = 0	

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

Generalize Traces

ca	cb	oa	ob	l
$1 = 1$	$1 = 1$	$0 = 0$	$0 = 0$	



Generalize Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>
1 = 1	1 = 1	0 = 0	0 = 0	
				2

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

Generalize Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>
$1 = 1$	$1 = 1$	$0 = 0$	$0 = 0$	$2 = B[1] $

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

Generalize Traces

ca	cb	oa	ob	l
$1 = 1$	$1 = 1$	$0 = 0$	$0 = 0$	$2 = B[cb] $

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

Generalize Traces

ca	cb	oa	ob	l
$1 = 1$	$1 = 1$	$0 = 0$	$0 = 0$	$2 = B[cb] $

A	c	e		c	e	c	e	c	e	c	e
B	c	e		c	e	c	e	c	e	c	e

Generalize Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>
1 = 1	1 = 1	0 = 0	0 = 0	
	2	2	0	$2 = B[cb] $

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

Generalize Traces

ca	cb	oa	ob	l
$1 = 1$	$1 = 1$	$0 = 0$	$0 = 0$	$2 = B[cb] $
	$2 = cb + 1$	2	$0 = 0$	

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

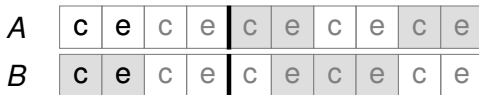
Generalize Traces

ca	cb	oa	ob	l
$1 = 1$	$1 = 1$	$0 = 0$	$0 = 0$	$2 = B[cb] $
	$2 = cb + 1$	$2 = oa + l$	$0 = 0$	

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

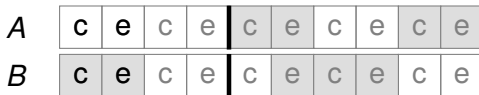
Generalize Traces

ca	cb	oa	ob	l
$1 = 1$	$1 = 1$	$0 = 0$	$0 = 0$	
	$2 = cb + 1$	$2 = oa + l$	$0 = 0$	$2 = B[cb] $



Generalize Traces

ca	cb	oa	ob	l
$1 = 1$	$1 = 1$	$0 = 0$	$0 = 0$	
	$2 = cb + 1$	$2 = oa + l$	$0 = 0$	$2 = B[cb] $
				2



Generalize Traces

ca	cb	oa	ob	l
$1 = 1$	$1 = 1$	$0 = 0$	$0 = 0$	
	$2 = cb + 1$	$2 = oa + l$	$0 = 0$	$2 = B[cb] $
				$2 = A[ca] - oa$

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

Generalize Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>
$1 = 1$	$1 = 1$	$0 = 0$	$0 = 0$	
	$2 = cb + 1$	$2 = oa + l$	$0 = 0$	$2 = B[cb] $
				$2 = A[ca] - oa$

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Generalize Traces

ca	cb	oa	ob	l
$1 = 1$	$1 = 1$	$0 = 0$	$0 = 0$	
	$2 = cb + 1$	$2 = oa + l$	$0 = 0$	$2 = B[cb] $
2		0	2	$2 = A[ca] - oa$

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

Generalize Traces

ca	cb	oa	ob	l
$1 = 1$	$1 = 1$	$0 = 0$	$0 = 0$	
	$2 = cb + 1$	$2 = oa + l$	$0 = 0$	$2 = B[cb] $
$2 = ca + 1$		$0 = 0$	2	$2 = A[ca] - oa$

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

Generalize Traces

ca	cb	oa	ob	l
$1 = 1$	$1 = 1$	$0 = 0$	$0 = 0$	$2 = B[cb] $
	$2 = cb + 1$	$2 = oa + l$	$0 = 0$	$2 = A[ca] - oa$
$2 = ca + 1$		$0 = 0$	$2 = ob + l$	

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

Generalize Traces

ca	cb	oa	ob	l
$1 = 1$	$1 = 1$	$0 = 0$	$0 = 0$	$2 = B[cb] $
	$2 = cb + 1$	$2 = oa + l$	$0 = 0$	$2 = A[ca] - oa$
$2 = ca + 1$		$0 = 0$	$2 = ob + l$	

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

Generalize Traces

ca	cb	oa	ob	l
$1 = 1$	$1 = 1$	$0 = 0$	$0 = 0$	$2 = B[cb] $
	$2 = cb + 1$	$2 = oa + l$	$0 = 0$	$2 = A[ca] - oa$
$2 = ca + 1$		$0 = 0$	$2 = ob + l$	1

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

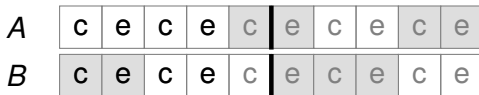
Generalize Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>
$1 = 1$	$1 = 1$	$0 = 0$	$0 = 0$	$2 = B[cb] $
	$2 = cb + 1$	$2 = oa + l$	$0 = 0$	$2 = A[ca] - oa$
$2 = ca + 1$		$0 = 0$	$2 = ob + l$	$1 = B[cb] - ob$

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

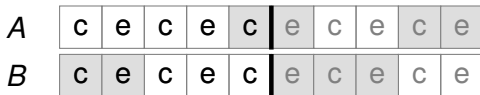
Generalize Traces

ca	cb	oa	ob	l
$1 = 1$	$1 = 1$	$0 = 0$	$0 = 0$	$2 = B[cb] - ob$
	$2 = cb + 1$	$2 = oa + l$	$0 = 0$	$2 = A[ca] - oa$
$2 = ca + 1$		$0 = 0$	$2 = ob + l$	$1 = B[cb] - ob$



Generalize Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>
$1 = 1$	$1 = 1$	$0 = 0$	$0 = 0$	$2 = B[cb] - ob$
	$2 = cb + 1$	$2 = oa + l$	$0 = 0$	$2 = A[ca] - oa$
$2 = ca + 1$		$0 = 0$	$2 = ob + l$	$1 = B[cb] - ob$



Generalize Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>
$1 = 1$	$1 = 1$	$0 = 0$	$0 = 0$	$2 = B[cb] - ob$
	$2 = cb + 1$	$2 = oa + l$	$0 = 0$	$2 = A[ca] - oa$
$2 = ca + 1$		$0 = 0$	$2 = ob + l$	$1 = B[cb] - ob$
	3	1	0	

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

Generalize Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>
$1 = 1$	$1 = 1$	$0 = 0$	$0 = 0$	$2 = B[cb] - ob$
	$2 = cb + 1$	$2 = oa + l$	$0 = 0$	$2 = A[ca] - oa$
$2 = ca + 1$		$0 = 0$	$2 = ob + l$	$1 = B[cb] - ob$
	$3 = cb + 1$	$1 = oa + l$	$0 = 0$	

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Generalize Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>
$1 = 1$	$1 = 1$	$0 = 0$	$0 = 0$	$2 = B[cb] - ob$
	$2 = cb + 1$	$2 = oa + l$	$0 = 0$	$2 = A[ca] - oa$
$2 = ca + 1$		$0 = 0$	$2 = ob + l$	$1 = B[cb] - ob$
	$3 = cb + 1$	$1 = oa + l$	$0 = 0$	

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Generalize Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>
$1 = 1$	$1 = 1$	$0 = 0$	$0 = 0$	$2 = B[cb] - ob$
	$2 = cb + 1$	$2 = oa + l$	$0 = 0$	$2 = A[ca] - oa$
$2 = ca + 1$		$0 = 0$	$2 = ob + l$	$1 = B[cb] - ob$
	$3 = cb + 1$	$1 = oa + l$	$0 = 0$	1

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Generalize Traces

ca	cb	oa	ob	l
$1 = 1$	$1 = 1$	$0 = 0$	$0 = 0$	$2 = B[cb] - ob$
	$2 = cb + 1$	$2 = oa + l$	$0 = 0$	$2 = A[ca] - oa$
$2 = ca + 1$		$0 = 0$	$2 = ob + l$	$1 = B[cb] - ob$
	$3 = cb + 1$	$1 = oa + l$	$0 = 0$	$1 = A[ca] - oa$

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

Generalize Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>
1 = 1	1 = 1	0 = 0	0 = 0	$2 = B[cb] - ob$
	2 = $cb + 1$	2 = $oa + l$	0 = 0	$2 = A[ca] - oa$
2 = $ca + 1$		0 = 0	2 = $ob + l$	1 = $ B[cb] - ob$
	3 = $cb + 1$	1 = $oa + l$	0 = 0	1 = $ A[ca] - oa$

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Generalize Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>
1 = 1	1 = 1	0 = 0	0 = 0	$2 = B[cb] - ob$
	2 = $cb + 1$	2 = $oa + l$	0 = 0	$2 = A[ca] - oa$
2 = $ca + 1$		0 = 0	2 = $ob + l$	1 = $ B[cb] - ob$
	3 = $cb + 1$	1 = $oa + l$	0 = 0	1 = $ A[ca] - oa$
3		0	1	

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Generalize Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>
1 = 1	1 = 1	0 = 0	0 = 0	$2 = B[cb] - ob$
	2 = <i>cb</i> + 1	2 = <i>oa</i> + <i>l</i>	0 = 0	$2 = A[ca] - oa$
2 = <i>ca</i> + 1		0 = 0	2 = <i>ob</i> + <i>l</i>	1 = $ B[cb] - ob$
	3 = <i>cb</i> + 1	1 = <i>oa</i> + <i>l</i>	0 = 0	1 = $ A[ca] - oa$
3 = <i>ca</i> + 1		0 = 0	1 = <i>ob</i> + <i>l</i>	

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

Generalize Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>
1 = 1	1 = 1	0 = 0	0 = 0	$2 = B[cb] - ob$
	2 = $cb + 1$	2 = $oa + l$	0 = 0	$2 = A[ca] - oa$
2 = $ca + 1$		0 = 0	2 = $ob + l$	1 = $ B[cb] - ob$
	3 = $cb + 1$	1 = $oa + l$	0 = 0	1 = $ A[ca] - oa$
3 = $ca + 1$		0 = 0	1 = $ob + l$	

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Generalize Traces

ca	cb	oa	ob	l
$1 = 1$	$1 = 1$	$0 = 0$	$0 = 0$	$2 = B[cb] - ob$
	$2 = cb + 1$	$2 = oa + l$	$0 = 0$	$2 = A[ca] - oa$
$2 = ca + 1$		$0 = 0$	$2 = ob + l$	$1 = B[cb] - ob$
	$3 = cb + 1$	$1 = oa + l$	$0 = 0$	$1 = A[ca] - oa$
$3 = ca + 1$		$0 = 0$	$1 = ob + l$	2

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

Generalize Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>
1 = 1	1 = 1	0 = 0	0 = 0	$2 = B[cb] - ob$
	2 = $cb + 1$	2 = $oa + l$	0 = 0	$2 = A[ca] - oa$
2 = $ca + 1$		0 = 0	2 = $ob + l$	$1 = B[cb] - ob$
	3 = $cb + 1$	1 = $oa + l$	0 = 0	$1 = A[ca] - oa$
3 = $ca + 1$		0 = 0	1 = $ob + l$	$2 = A[ca] - oa$

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Generalize Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>
1 = 1	1 = 1	0 = 0	0 = 0	2 = B[cb] - ob
	2 = cb + 1	2 = oa + l	0 = 0	2 = A[ca] - oa
2 = ca + 1		0 = 0	2 = ob + l	1 = B[cb] - ob
	3 = cb + 1	1 = oa + l	0 = 0	1 = A[ca] - oa
3 = ca + 1		0 = 0	1 = ob + l	2 = A[ca] - oa

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Generalize Traces

<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>
1 = 1	1 = 1	0 = 0	0 = 0	$2 = B[cb] - ob$
	2 = <i>cb</i> + 1	2 = <i>oa</i> + <i>l</i>	0 = 0	$2 = A[ca] - oa$
2 = <i>ca</i> + 1		0 = 0	2 = <i>ob</i> + <i>l</i>	1 = $ B[cb] - ob$
	3 = <i>cb</i> + 1	1 = <i>oa</i> + <i>l</i>	0 = 0	1 = $ A[ca] - oa$
3 = <i>ca</i> + 1		0 = 0	1 = <i>ob</i> + <i>l</i>	2 = $ A[ca] - oa$
4	4	0	0	

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Generalize Traces

ca	cb	oa	ob	l
$1 = 1$	$1 = 1$	$0 = 0$	$0 = 0$	$2 = B[cb] - ob$
	$2 = cb + 1$	$2 = oa + l$	$0 = 0$	$2 = A[ca] - oa$
$2 = ca + 1$		$0 = 0$	$2 = ob + l$	$1 = B[cb] - ob$
	$3 = cb + 1$	$1 = oa + l$	$0 = 0$	$1 = A[ca] - oa$
$3 = ca + 1$		$0 = 0$	$1 = ob + l$	$2 = A[ca] - oa$
$4 = ca + 1$	$4 = cb + 1$	$0 = 0$	$0 = 0$	

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

Generalize Traces

ca	cb	oa	ob	l
$1 = 1$	$1 = 1$	$0 = 0$	$0 = 0$	$2 = B[cb] - ob$
	$2 = cb + 1$	$2 = oa + l$	$0 = 0$	$2 = A[ca] - oa$
$2 = ca + 1$		$0 = 0$	$2 = ob + l$	$1 = B[cb] - ob$
	$3 = cb + 1$	$1 = oa + l$	$0 = 0$	$1 = A[ca] - oa$
$3 = ca + 1$		$0 = 0$	$1 = ob + l$	$2 = A[ca] - oa$
$4 = ca + 1$	$4 = cb + 1$	$0 = 0$	$0 = 0$	
\vdots	\vdots	\vdots	\vdots	\vdots

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

Generalize Traces

ca	cb	oa	ob	l
$1 = 1$	$1 = 1$	$0 = 0$	$0 = 0$	$2 = B[cb] - ob$
	$2 = cb + 1$	$2 = oa + l$	$0 = 0$	$2 = A[ca] - oa$
$2 = ca + 1$		$0 = 0$	$2 = ob + l$	$1 = B[cb] - ob$
	$3 = cb + 1$	$1 = oa + l$	$0 = 0$	$1 = A[ca] - oa$
$3 = ca + 1$		$0 = 0$	$1 = ob + l$	$2 = A[ca] - oa$
$4 = ca + 1$	$4 = cb + 1$	$0 = 0$	$0 = 0$	
\vdots	\vdots	\vdots	\vdots	\vdots

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

Generalize Traces

ca	cb	oa	ob	l
1	1	0	0	$ B[cb] - ob$
	$cb + 1$	$oa + l$	0	$ A[ca] - oa$
$ca + 1$		0	$ob + l$	$ B[cb] - ob$
	$cb + 1$	$oa + l$	0	$ A[ca] - oa$
$ca + 1$		0	$ob + l$	$ A[ca] - oa$
$ca + 1$	$cb + 1$	0	0	
\vdots	\vdots	\vdots	\vdots	\vdots

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

Generalize Traces

ca	cb	oa	ob	l
1	1	0	0	
	$cb + 1$	$oa + l$	0	$ B[cb] - ob$
$ca + 1$		0	$ob + l$	$ A[ca] - oa$
	$cb + 1$	$oa + l$	0	$ B[cb] - ob$
$ca + 1$		0	$ob + l$	$ A[ca] - oa$
$ca + 1$	$cb + 1$	0	0	$ A[ca] - oa$
\vdots	\vdots	\vdots	\vdots	\vdots

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

Generalize Traces

Name	<i>ca</i>	<i>cb</i>	<i>oa</i>	<i>ob</i>	<i>l</i>
	1	1	0	0	
		<i>cb</i> + 1	<i>oa</i> + <i>l</i>	0	$ B[cb] - ob$
<i>ca</i> + 1			0	<i>ob</i> + <i>l</i>	$ A[ca] - oa$
		<i>cb</i> + 1	<i>oa</i> + <i>l</i>	0	$ B[cb] - ob$
<i>ca</i> + 1			0	<i>ob</i> + <i>l</i>	$ A[ca] - oa$
<i>ca</i> + 1	<i>cb</i> + 1		0	0	$ A[ca] - oa$
⋮	⋮	⋮	⋮	⋮	⋮

<i>A</i>	c	e	c	e	c	e	c	e	c	e
<i>B</i>	c	e	c	e	c	e	c	e	c	e

Generalize Traces

Name	ca	cb	oa	ob	l
INIT	1	1	0	0	
		$cb + 1$	$oa + l$	0	$ B[cb] - ob$
	$ca + 1$		0	$ob + l$	$ A[ca] - oa$
		$cb + 1$	$oa + l$	0	$ B[cb] - ob$
	$ca + 1$		0	$ob + l$	$ A[ca] - oa$
	$ca + 1$	$cb + 1$	0	0	$ A[ca] - oa$
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

Generalize Traces

Name	ca	cb	oa	ob	l
BLEN	1	1	0	0	$ B[cb] - ob$
		$cb + 1$	$oa + l$	0	$ A[ca] - oa$
BLEN	$ca + 1$		0	$ob + l$	$ B[cb] - ob$
		$cb + 1$	$oa + l$	0	$ A[ca] - oa$
	$ca + 1$		0	$ob + l$	$ A[ca] - oa$
	$ca + 1$	$cb + 1$	0	0	
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

Generalize Traces

Name	ca	cb	oa	ob	l
	1	1	0	0	
		$cb + 1$	$oa + l$	0	$ B[cb] - ob$
ALEN	$ca + 1$		0	$ob + l$	$ A[ca] - oa$
		$cb + 1$	$oa + l$	0	$ B[cb] - ob$
ALEN	$ca + 1$		0	$ob + l$	$ A[ca] - oa$
ALEN	$ca + 1$	$cb + 1$	0	0	$ A[ca] - oa$
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

Generalize Traces

Name	ca	cb	oa	ob	l
	1	1	0	0	
ASBN		$cb + 1$	$oa + l$	0	$ B[cb] - ob$
	$ca + 1$		0	$ob + l$	$ A[ca] - oa$
ASBN		$cb + 1$	$oa + l$	0	$ B[cb] - ob$
	$ca + 1$		0	$ob + l$	$ A[ca] - oa$
	$ca + 1$	$cb + 1$	0	0	$ A[ca] - oa$
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

Generalize Traces

Name	ca	cb	oa	ob	l
	1	1	0	0	
		$cb + 1$	$oa + l$	0	$ B[cb] - ob$
ANBS	$ca + 1$		0	$ob + l$	$ A[ca] - oa$
		$cb + 1$	$oa + l$	0	$ B[cb] - ob$
ANBS	$ca + 1$		0	$ob + l$	$ A[ca] - oa$
	$ca + 1$	$cb + 1$	0	0	$ A[ca] - oa$
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

Generalize Traces

Name	ca	cb	oa	ob	l
	1	1	0	0	
		$cb + 1$	$oa + l$	0	$ B[cb] - ob$
	$ca + 1$		0	$ob + l$	$ A[ca] - oa$
		$cb + 1$	$oa + l$	0	$ B[cb] - ob$
	$ca + 1$		0	$ob + l$	$ A[ca] - oa$
ANBN	$ca + 1$	$cb + 1$	0	0	$ A[ca] - oa$
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

Generalize Traces

Name	ca	cb	oa	ob	l
INIT	1	1	0	0	
BLEN					$ B[cb] - ob$
ASBN		$cb + 1$	$oa + l$	0	
ALEN					$ A[ca] - oa$
ANBS	$ca + 1$		0	$ob + l$	
BLEN					$ B[cb] - ob$
ASBN		$cb + 1$	$oa + l$	0	
ALEN					$ A[ca] - oa$
ANBS	$ca + 1$		0	$ob + l$	
ALEN					$ A[ca] - oa$
ANBN	$ca + 1$	$cb + 1$	0	0	
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots

A	c	e	c	e	c	e	c	e	c	e
B	c	e	c	e	c	e	c	e	c	e

Synthesize Control Flow

INIT → BLEN → ASBN → ALEN → ANBS → BLEN → ... → ALEN → YES

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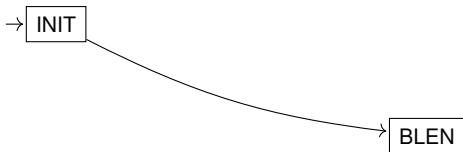
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→ INIT

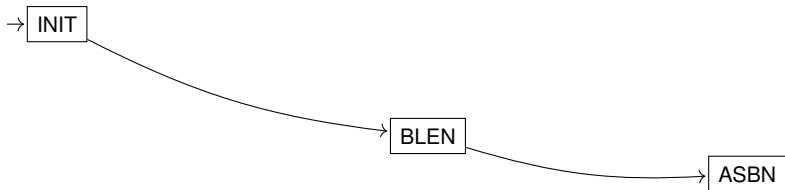
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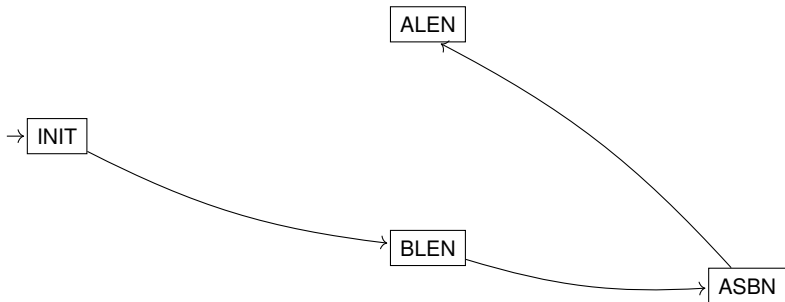
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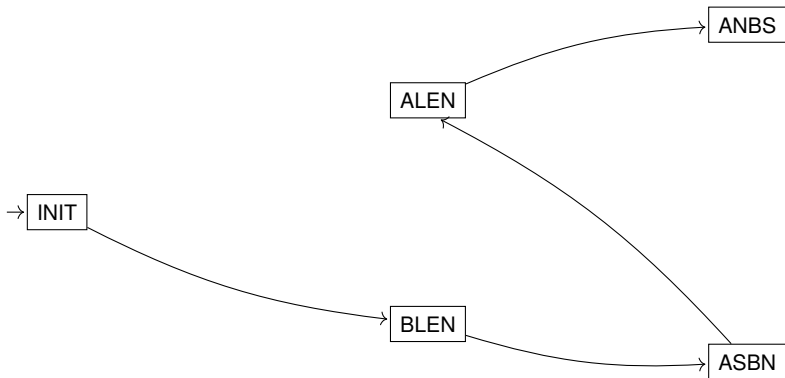
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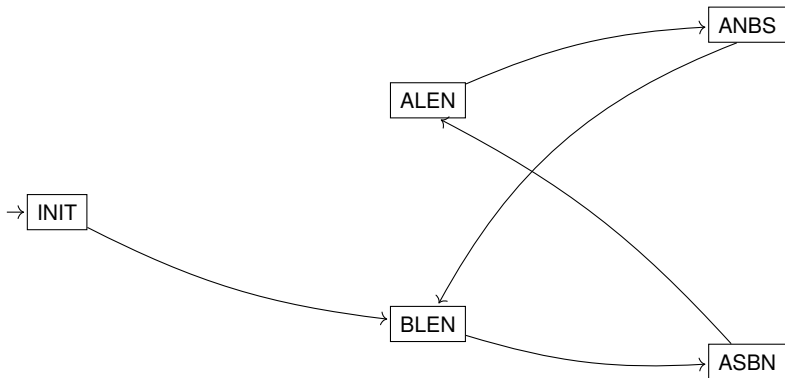
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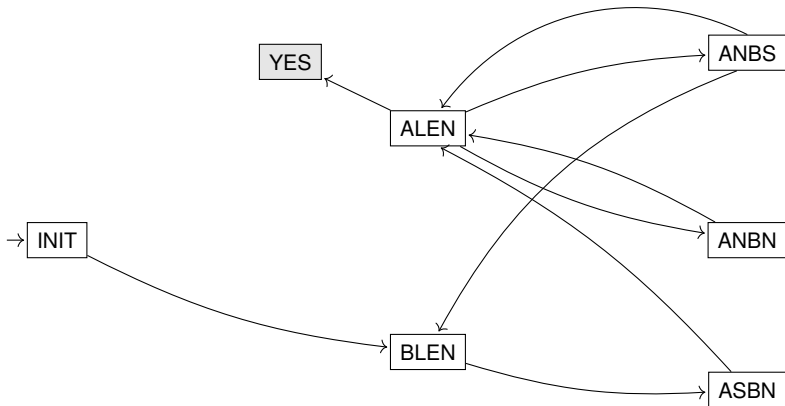
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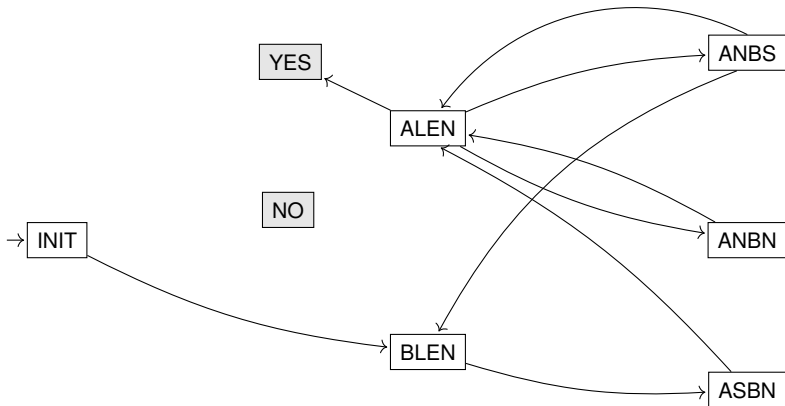
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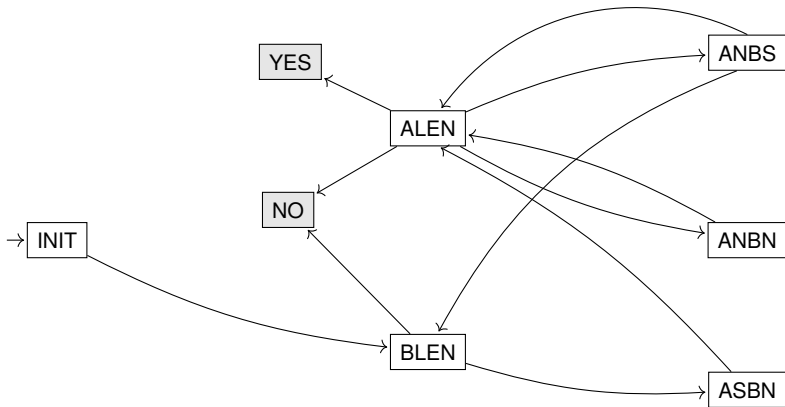
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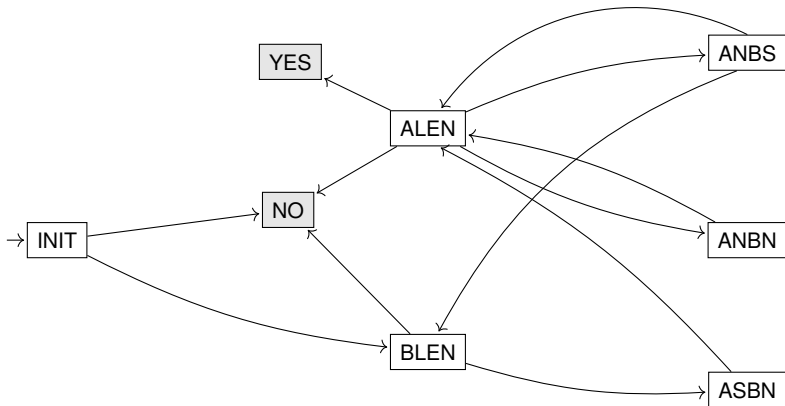
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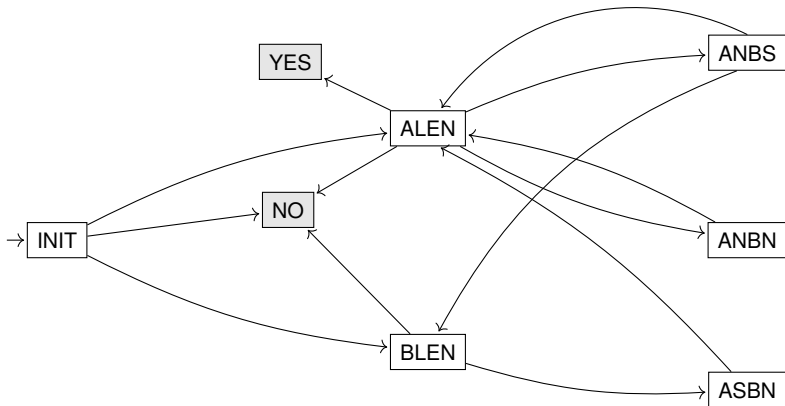
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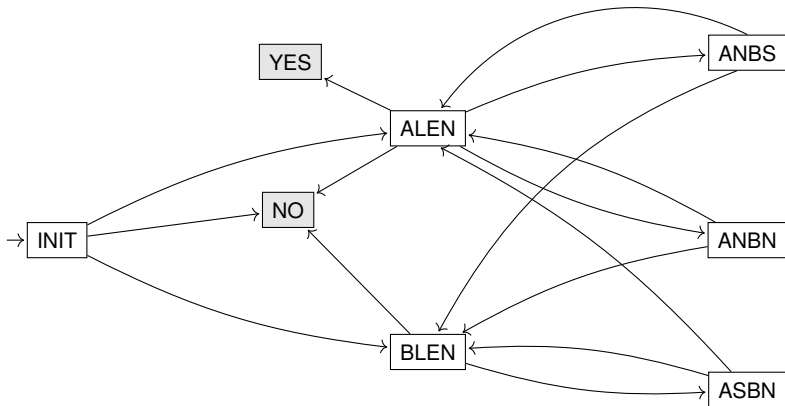
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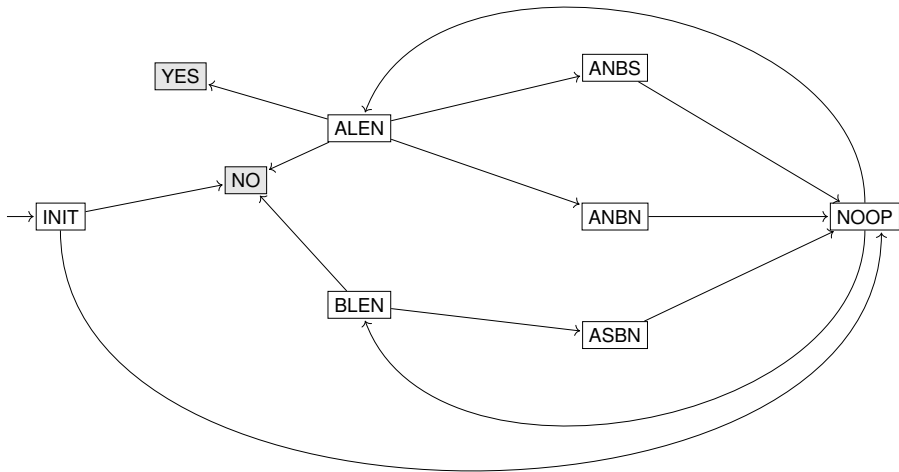


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Synthesize Control Flow



Add Edge Predicates

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ALEN \rightarrow { NO, YES, ANBS, ANBN }

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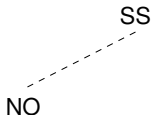
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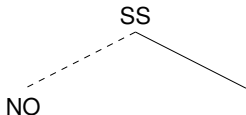
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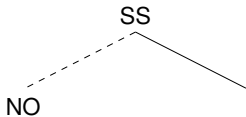
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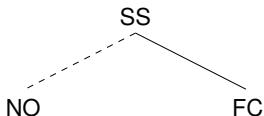
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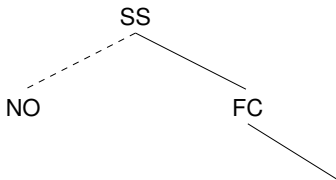
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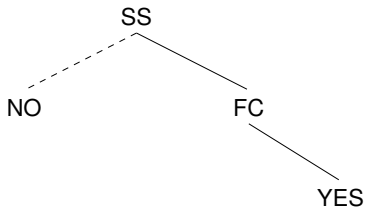
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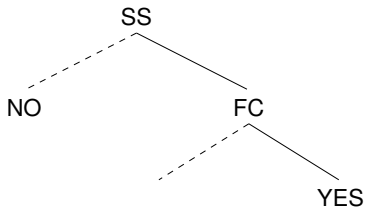
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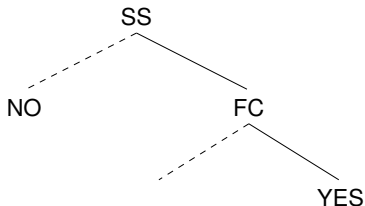
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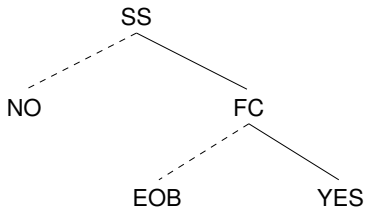
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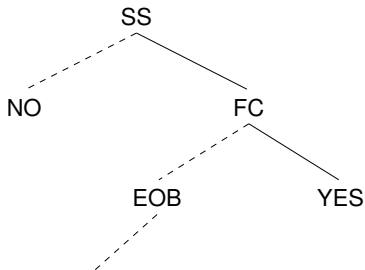
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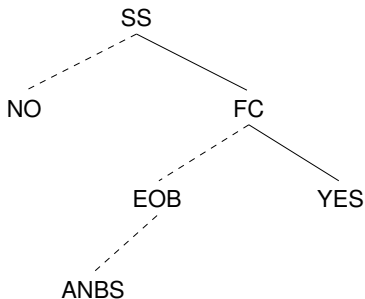
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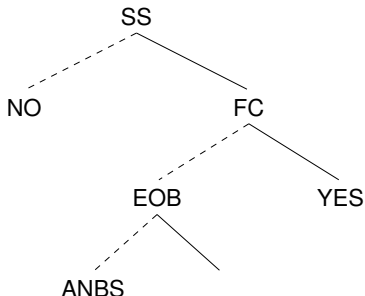
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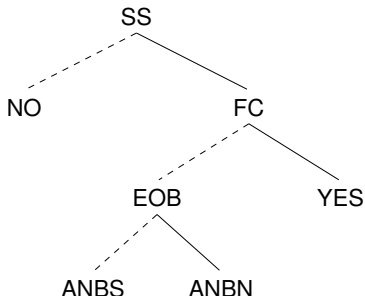
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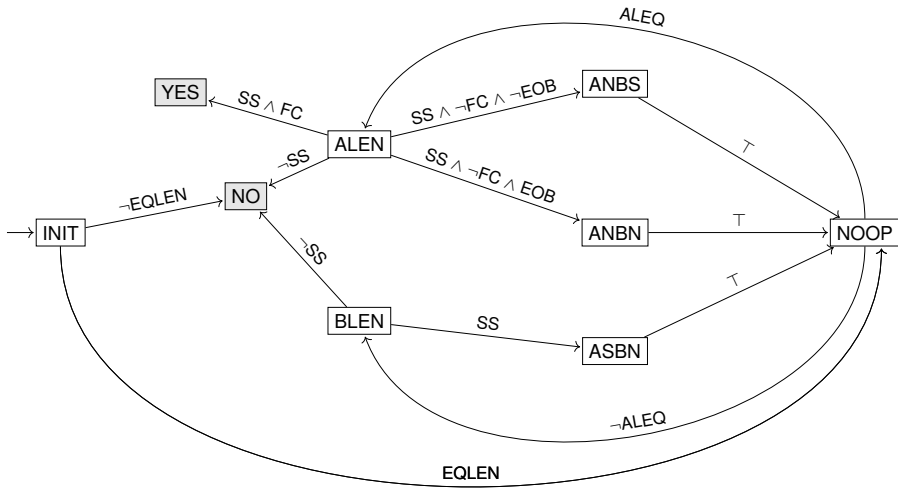
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Complete Program



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1-to-1 correspondence of program states & operations